Buckling Analysis of Laminated Composite Plates with a Central Hole

By-

Pratyasha Patnaik

Roll No 110CE0050

Bachelor in Technology in Civil Engineering

Under the guidance of-

Prof. A.V.Asha

Department Of Civil Engineering

National Institute of Technology Rourkela

CERTIFICATE

This is to certify that the thesis entitled "**Buckling Analysis of Laminated Composite Plates with a central Hole**" submitted by **Miss Pratyasha Patnaik** in partial fulfillment of the requirement for the award of **Bachelor of Technology** Degree in **Civil Engineering** at **National Institute of Technology Rourkela** is an authentic work carried out by her under my guidance and supervision.

To the best of my knowledge, the matter presented in this thesis has not been presented in any other university/college for any other degree or diploma.

Date: May 03, 2014

Place: Rourkela

Prof. A.V.Asha
Department of Civil Engineering
NIT ROURKELA

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Pratyasha Patnaik

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ABSTRACT

Laminated composite plates are made up of plates consisting of layers bonded together and made up of materials chemically different from each other but combined macroscopically. These have an application in aircrafts, railway coaches, bridges et cetera because they are easy to handle, have got improved properties and the cost of their fabrication is low. But their failure can lead to catastrophic disasters. And generally the failure of these structures is due to the combined effect of excessive stresses on it and buckling. Hence the buckling behavior of these kinds of plates should be analyzed properly. Buckling behavior of laminated composite plates subjected to in-plane loads is an important aspect in the preliminary design of aircraft and launch vehicle components. Holes are provided either in the center or elsewhere in the laminar plates for the purpose of pipes for electric cables or other purposes. Due to the presence of holes in the plates, the stress concentration is near to the holes and the stiffness of the plates is reduced. Hence the study is important in order to know the buckling behavior of such plates. The composite materials are advantageous to be used in comparison to conventional materials due to its excellent mechanical properties such as its durability, low density and corrosion-resistant characteristics.

In this study, the effect of cut-out, its shape, different boundary conditions, length/thickness ratio, stacking sequence, and ply orientation has been studied. Analysis was carried out with laminated composite plates with circular, square and triangular cut-outs. Results show the effect of different cut-out shapes, different boundary conditions, orientation of layers and length/thickness ratio on the buckling load.

CHAPTER 1

INTRODUCTION

1.1 INTRODUCTION

Most of the engineering structures fail due to excessive stresses developed in them due to the external loadings on them and due to buckling. In the present study, only rectangular thin plates have been taken into account. When a flat plate is subjected to compressive load, it initially remains flat and stays in equilibrium condition. But as the compressive force increases to a certain amount, the plate becomes unstable and its configuration changes from flat to non-flat. The load at which the plate leaves its equilibrium condition and becomes unstable is known as “Critical Buckling Load”.

A composite material is composed of two or more materials and possesses the properties which could not have been achieved from any of its constituent materials alone. In such materials the main load bearing members are the fibres. The matrix has low modulus and high elongation and it provides flexibility to the structure, keeps the fibres in position and protects them from the external forces of the environment.

Conventional products may have one property advantageous to the strength of the structures. But now-a-days there is requirement for many properties which can assure the stability of the structure completely. Hence the use of composite structures has accelerated due to the combination of properties it possesses or to be precise, its heterogeneous nature. Properties of composites are due to its constituent materials, their distribution and their orientation which altogether gives an unusual combination of properties.

Laminated composites have wide use in mechanical and aerospace applications due to their high specific stiffness and high specific strength. Fiber-reinforced composites usually exist in the form of thin plates. They are most of the time subjected to compressive loads which when it reaches critical buckling load has a possibility of failure. Hence the buckling behavior of the composites has been a major concern for the researchers.

In this study, the effect of cut-out, their shapes, different boundary conditions, different length/thickness ratio, different stacking sequence and different ply orientation on the buckling load has been investigated analytically by the use of the software ANSYS.

1.2 SCOPE OF STUDY

Composite materials have widespread application in mechanical, aerospace, biomedical engineering fields because it is easy to handle, has got good mechanical properties and the cost of its fabrication is also low. The failure of composites is generally due to buckling. Hence the buckling behavior needs to be properly analyzed. Cutouts are often required in structural components due to functional requirements, and to produce lighter and more efficient structures. Most stability studies of composite plates with cutout have focused on square plates under simply supported conditions to minimize the mathematical formulations. The aim of this project is to study the effect on buckling load of square and rectangular laminated plates with and without a hole subjected to compressive loads with different boundary conditions.

CHAPTER 2

LITERATURE REVIEW

2.1 LITERATURE REVIEW

Laminated composites are used generally as thin plates, and under compressive load the load carrying capacity is investigated by most of the researchers. Thus far, there has been research on the laminated structures which find applications in aerospace, biomedical, civil, and marine and mechanical engineering because of the improved properties they have and its cost-effectiveness and because of the ease with which they can be handled.

Hu and Lin in 1995 [5] studied the buckling resistance of symmetrically laminated plates with a given material system and subjected to uniaxial compression. The research was done with plates having different plate thicknesses, aspect ratios, central circular cutouts and different end conditions. Due to these variations, the optimal fiber orientations and the associated optimal buckling loads of symmetrically laminated plates were investigated.

Due to the importance of buckling analysis of composite structures in various industrial applications, a comparative study of buckling behavior of composite plates was done by Darvizeh et al in 2004 [7]. Mathematical modelling developed in this work for generally laminated plates was based on generalized differential quadrature rule (GDQR) and Rayleigh-Ritz method. The buckling load was analyzed using both the methods and then compared. The comparison shown in form of tables showed the efficiency of GDQR.

Xie and Ni in 2005 [8] studied the buckling of laminated composite plates with internal supports. Both the higher-order shear deformation theory and *pb*-2 Ritz displacement functions, corresponding to an arbitrary edge support were used in this paper. The buckling load was investigated under biaxial compressive loading. Numerical results emphasized the effect of angle of lamination, boundary conditions, aspect ratio, and internal supports on critical buckling load.

Ni et al in 2005 [9] did a buckling analysis for rectangular laminated composite plates with biaxial compressive load acting on them. The higher-order shear deformation theory was used and a special displacement function, which could express an arbitrary edge support, was introduced into the Rayleigh–Ritz method. Furthermore in this paper, the buckling modes were determined.

Khalili et al also in 2005 [10] developed a new analytical method to investigate the response of laminated composite plates subjected to static and dynamic loading. The modal forms were presented in terms of double Fourier series. The derivatives of the double Fourier series were legitimized using Stoke's transformation.

The influence of boundary conditions on the buckling load for rectangular plates of various cutout shape, length/thickness ratio, and ply orientation was examined by Baba in 2007 [11]. Boundary conditions considered were clamped, pinned and their various combinations. The plates were subjected to in-plane compression load. The results of experimentation were validated using numerical analysis by ANSYS.

An exact solution for buckling of simply supported symmetrical cross-ply composite rectangular plates under a linearly varying boundary load was presented by Zhong and Gu in 2007 [12]. It was developed based on the first-order shear deformation theory for moderately thick laminated plates. Buckling loads of cross-ply rectangular plates with various aspect ratios were obtained and the effects of load intensity variation and layup configuration on the buckling load were also investigated. The results were verified using the computer code ABAQUS.

Topal and Uzman in 2008 [13] analysed the buckling of rectangular composite laminates with circular holes under planar static loadings. First order shear deformation theory and the variational energy method were used for the study. A nine-node Lagrangian finite element method was used for finding critical loads. The effects due to cut-out size, plate thickness ratio, material modulus ratio, ply lamination geometry, loading types, and boundary conditions on buckling load were investigated.

Qablan et al in 2009 [14] evaluated the effect of cutout size, cutout location, fiber orientation angle and type of loading on the buckling load of square cross-ply laminated plates with circular cutouts. Three types of in-plane loading were considered; namely, uniaxial compression, biaxial compression and shear loading.

Komur et al in 2010 [15] carried out a numerical buckling analysis on a woven–glass–polyester laminated composite plate with a circular/elliptical hole. The laminated composite plates were arranged as symmetric cross-ply [(0_/90_) 2] s and angle ply [(15_/_75_) 2] s, [(30_/_60_) 2] s, [(45_/_45_) 2] s. Finite element method (FEM) was applied to perform research on various plates based on the shape and position of the elliptical hole.

Gaira et al in 2012 [16] worked out the buckling load factors for laminated composites with different aspect ratio, d/b ratio and d/D ratio. They found that the presence of cut-outs lowered the buckling load factors. Also the factor increased with increase of aspect ratio up to 1.11. The load was inversely proportional to d/b ratio up to 0.15 and also inversely proportional to d/D ratio up to 0.25.

Yousef et al in 2012 [17] presented an experimental study of the behavior of woven glass fiber/epoxy composite laminated panels with ply orientation (+45º/-45º/+45º) s under compression. Compression tests were performed on plates with and without holes. The results indicated that the plates without cut-out exhibited higher fracture load and energy absorption than plates with cut-out.

2.2 OBJECTIVE OF PRESENT STUDY

Laminated composite plates are made up of plates consisting of layers bonded together and made up of materials chemically different from each other but combined macroscopically. These have an application in aircrafts, railway coaches, bridges etc. But their failure can lead to catastrophic disasters. And generally the failure of these structures is due to the combined effect of excessive stresses on it and buckling. Hence the buckling behavior of these kinds of plates should be analyzed properly. These plates may have holes placed centrally or otherwise for the purpose of pipes for electric cables or other purposes. This may reduce the stiffness of the plate and create stress concentrations in the region of the holes. Hence the study is important in order to know the buckling behavior of such plates.

The present study is done to analyze the effect on buckling load due to changes in modulus ratio, length/thickness ratio, boundary conditions, stacking sequence and ply orientation. The effect of cut-out and its shape are also studied.

CHAPTER 3

THEORY

3.1 THEORETICAL FORMULATION

Two planes, i.e xz, yz planes and two edge conditions on each boundary of the plate are involved in the buckling of a plate. There is a basic difference in the buckling characteristics for column and plates. For column, the buckling load is equal to its failure load because once a column buckles, it cannot resist any more load. But plates are able to resist 10-15 times the value of primary buckling load because they are supported at the edges and they do not fail so easily.

3.2 THEORY OF BENDING OF THIN PLATES

The theory of bending for thin plates is similar to the theory for beams. In pure bending of beams, "the stress distribution is obtained by the assumption that cross-sections of the bar remain plane before and after bending and rotate only with respect to their neutral axes so as to be always normal to the deflection curve." For a thin plate, bending in two perpendicular directions occur. A rectangular plate element is as shown:

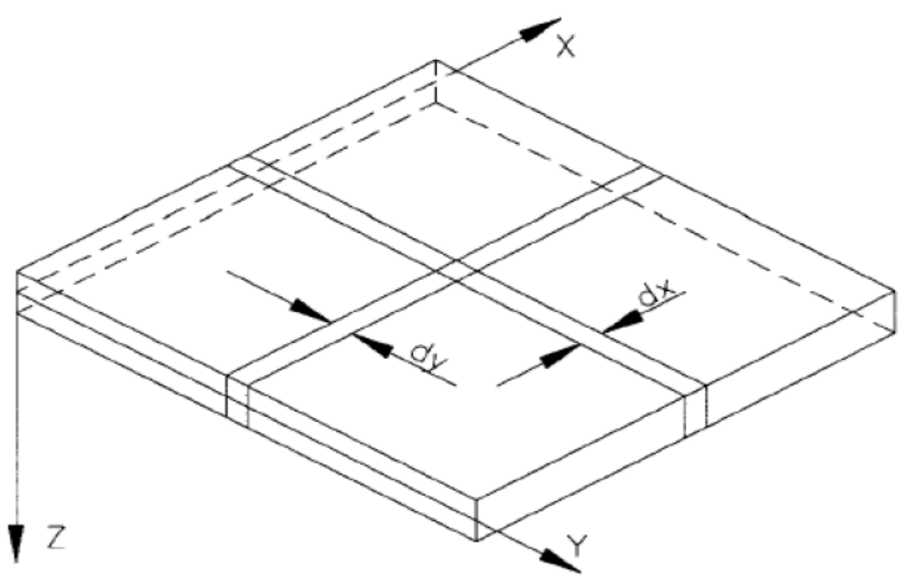


Fig 1: Thin Plate Notation

The basic assumptions of elastic plate bending are:

1. Plates are perfectly flat and are of uniform thickness.

2. The thickness of the plate is very small compared with its length and breadth.

3. Deflections are small, i.e., smaller or equal to 1/2 of the thickness.

4. The middle plane of the plate remains on the neutral surface and does not undergo any elongation.

5. The lateral sides of the differential element, in the above figure, remain plane during bending and rotate so as to remain normal to the deflection surface. Hence, the stresses and strains increase as the distance from the neutral axis increases.

6. The applied loads are resisted by the bending and the twisting of the plates. The effect of shearing forces is neglected.

3.3 THEORY OF BENDING OF COMPOSITE PLATES

Composite materials consist of two or more materials producing desirable properties that cannot be achieved with any of the constituents alone. Fiber-reinforced composite materials contain high strength. Fibers are the principal load carrying members, and the matrix material keeps the fibers intact and protects it from the environment. Each ply is a thin plate which together with other plies forms a composite. The orientation of each ply is arbitrary, and the layup sequence is so adjusted to achieve the desirable properties.

Each thin layer is called a lamina. A lamina is a macro unit of material whose material properties are determined by appropriate laboratory tests. Structural elements are formed by stacking the laminas to achieve desired properties. Fiber orientation in each lamina and stacking sequence of the layers are so chosen to achieve desired strength and stiffness.

3.4 GOVERNING EQUATION

The building block for a laminated composite is a lamina. The properties of the laminate are derived from the properties of its constituent materials. But to get the desired strength and stiffness, the proper materials with proper orientation has to be put together. Hence before that, knowledge of the stress and strain through the laminate thickness is necessary.

The following assumptions are made regarding the behavior of the laminate:

1. It is made up of perfectly bonded laminas.

2. The bonds are infinitesimally thin and there is no relative slipping between the laminas. This implies that there is a continuous displacement across the lamina boundaries. As a result, the laminate behaves like a lamina with special properties.

3. A line originally straight and perpendicular to the middle surface of the laminate remains straight and perpendicular to the middle surface even after buckling.

4. The strain perpendicular to the middle surface of the laminate is neglected.

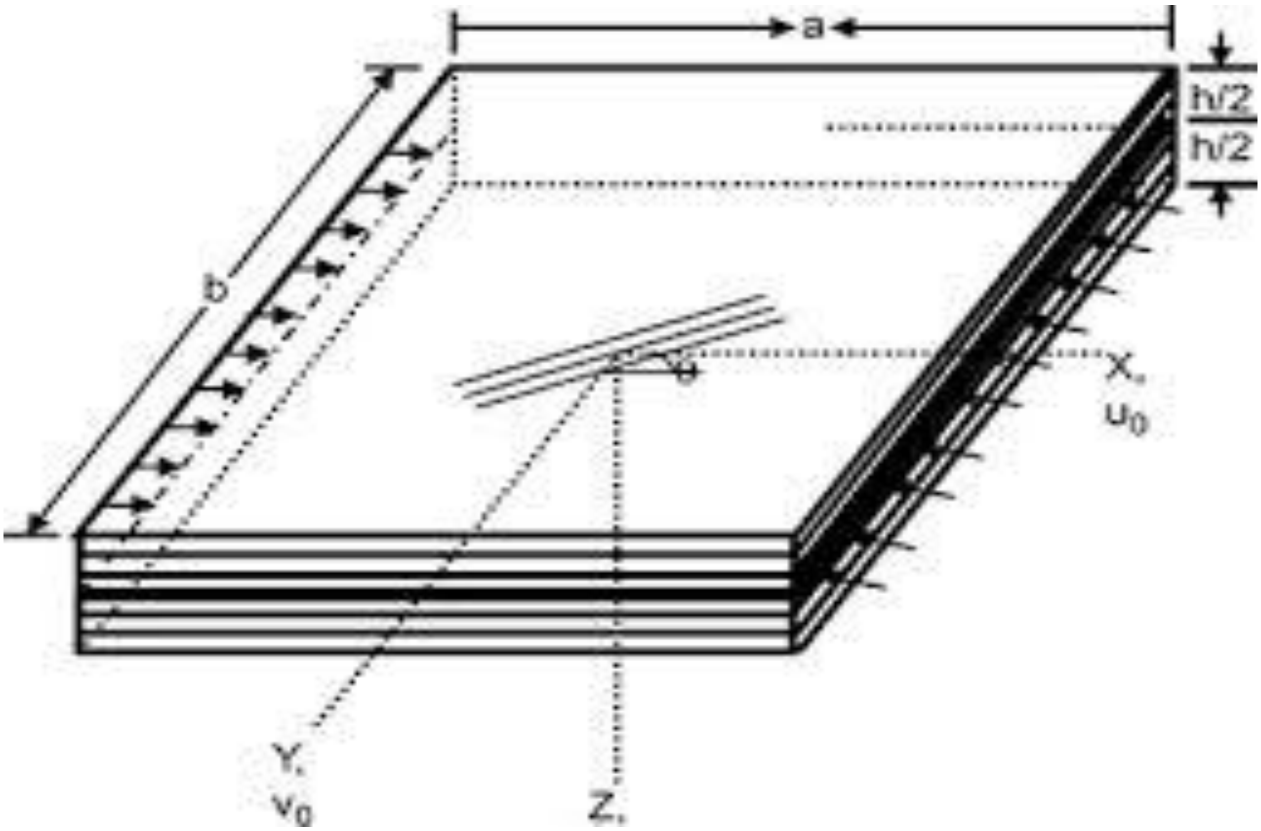


Fig 2: Laminated composite plate under in-plane compression

The differential equations of motion are obtained by taking a differential element of the panel, as shown in figure 2. This figure shows an element with internal forces like membrane forces N_x ,N_y and N_{xy}, shearing forces(Q_x and Q_y) and the moment resultants($M_x,\ M_y\ and\ M_{xy}$).

The governing differential equations of equilibrium for a shear deformable doubly curved panel subjected to external in-plane loading can be expressed as (Chandrashekhara[4] , Sahu and Dutta) [6]:

$$\frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} = P_1 \frac{\partial^2 u}{\partial t^2} + P_2 \frac{\partial^2 \theta_x}{\partial t^2}$$

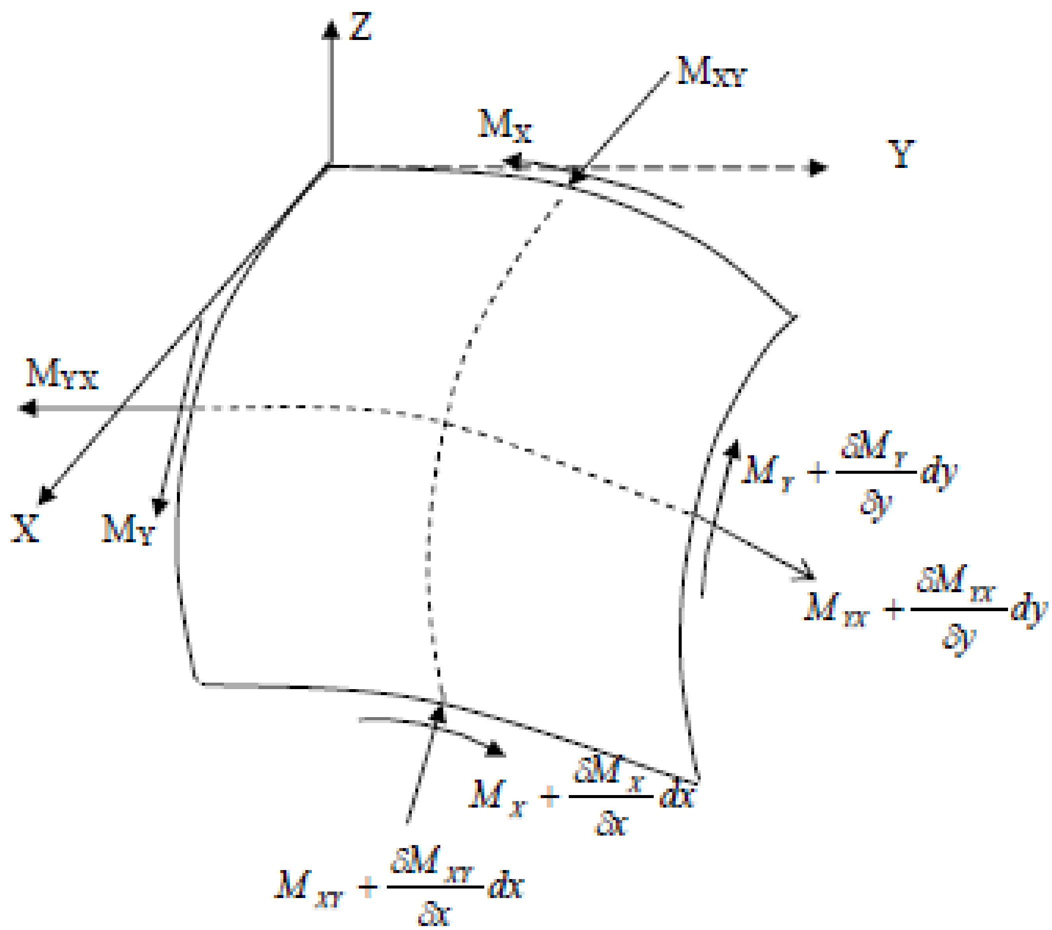
$$\frac{\partial N_{xy}}{\partial x} + \frac{\partial N_y}{\partial y} = P_1 \frac{\partial^2 v}{\partial t^2} + P_2 \frac{\partial^2 \theta_y}{\partial t^2}$$

$$\frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} + N_x^0 \frac{\partial^2 w}{\partial x^2} + N_y^0 \frac{\partial^2 w}{\partial y^2} = P_1 \frac{\partial^2 w}{\partial t^2}$$

$$\frac{\partial M_x}{\partial x} + \frac{\partial M_{xy}}{\partial y} - Q_x = P_1 \frac{\partial^2 \theta_x}{\partial t^2} + P_2 \frac{\partial^2 u}{\partial t^2}$$

$$\frac{\partial M_{xy}}{\partial x} + \frac{\partial M_y}{\partial y} - Q_y = P_1 \frac{\partial^2 \theta_y}{\partial t^2} + P_2 \frac{\partial^2 v}{\partial t^2} \tag{3.1}$$

N_x^0 and N_y^0 are the external loading in the X and Y directions respectively.



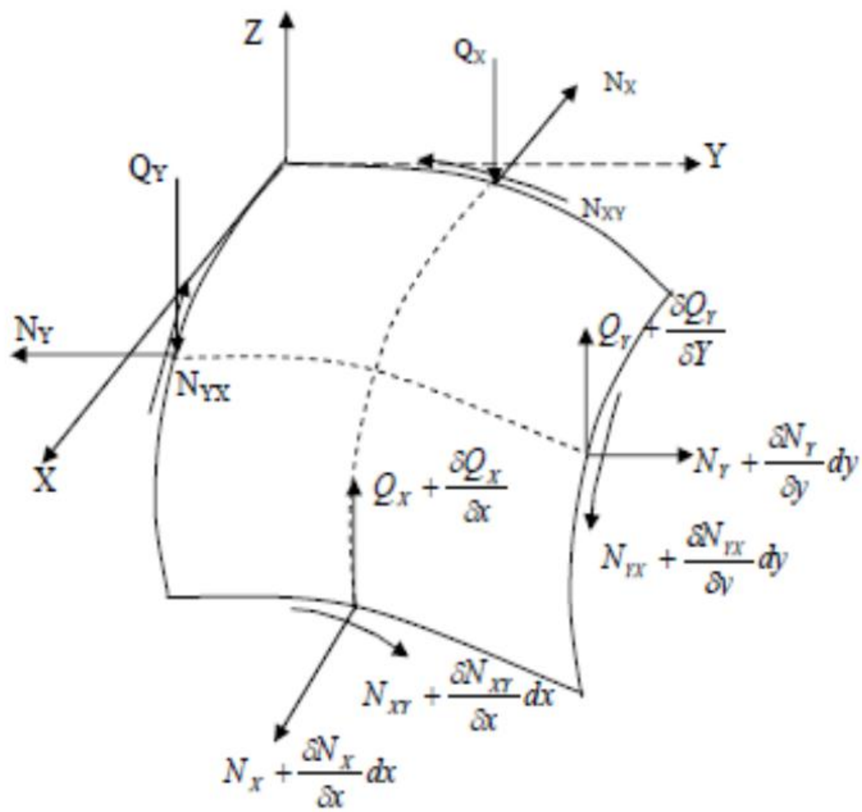


Fig 3: Element of a shell panel

The constants R_x, R_y and R_{xy} are the radii of curvature in the x and y directions and the radius of twist.

$$(P_1, P_2, P_3) = \sum_{k=1}^{n} \int_{Z_{k-1}}^{Z_k} (\rho)_k (1, z, z^2) dz \tag{3.2}$$

where n= number of layers of the laminated composite panel and $(\rho)_k$ = mass density of k_{th} layer from the mid-plane.

In the present study, only flat plates have been analyzed. Hence R_x, R_y and R_{xy} are all infinity.

3.5 CONSTITUTIVE EQUATIONS:

The basic laminated composite panel is considered to be composed of composite material laminates (typically thin layers). The matrix material keeps the fibers intact in them. Each layer may be regarded as being homogeneous and orthotropic. The laminated fiber reinforced shell is assumed to consist of a number of thin laminates as shown in figure 4. The principle material axes are indicated by 1 and 2 and moduli of elasticity of a lamina along these directions are E_{11} and E_{22} respectively.

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \\ \tau_{xz} \\ \tau_{yz} \end{bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 & 0 & 0 \\ Q_{12} & Q_{22} & 0 & 0 & 0 \\ 0 & 0 & Q_{66} & 0 & 0 \\ 0 & 0 & 0 & Q_{44} & 0 \\ 0 & 0 & 0 & 0 & Q_{55} \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \\ \gamma_{xz} \\ \gamma_{yz} \end{bmatrix} \tag{3.3}$$

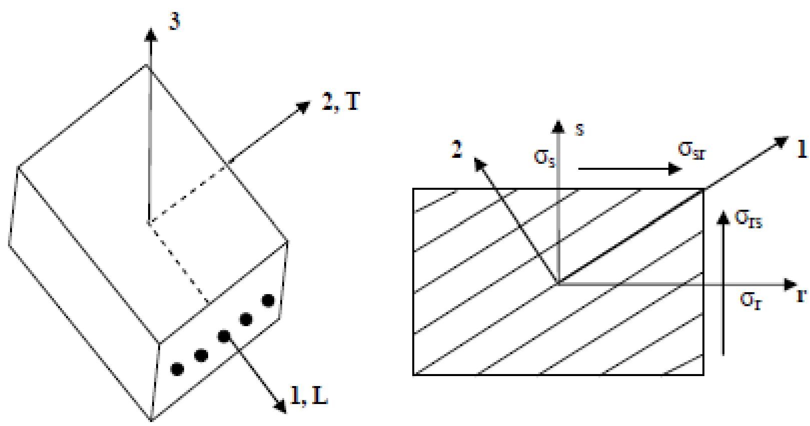


Fig 4: Laminated shell element showing principal axes and laminate directions

Where

$$Q_{11} = \frac{E_{11}}{(1-v_{12}v_{21})}$$

$$Q_{12} = \frac{E_{11}v_{21}}{(1-v_{12}v_{21})}$$

$$Q_{21}= \frac{E_{22}}{(1-v_{12}v_{21})}$$

$$Q_{22}= \frac{E_{22}}{(1-v_{12}v_{21})} \tag{3.4}$$

$$Q_{66}= G_{12}$$

$$Q_{44}= \mathrm{k}G_{13}$$

$$Q_{55}= kG_{23}$$

The on – axis elastic constant matrix corresponding to the fiber direction is given by

$$[\bar{Q}_{ij}] = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} & 0 & 0 \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} & 0 & 0 \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} & 0 & 0 \\ 0 & 0 & 0 & \bar{Q}_{44} & \bar{Q}_{45} \\ 0 & 0 & 0 & \bar{Q}_{45} & \bar{Q}_{55} \end{bmatrix} \tag{3.5}$$

If the major and minor Poisson's ratio are ν_{12} and ν_{21}, then using reciprocal relation one obtains the following well known expression

$$\frac{\vartheta_{12}}{E_{11}} = \frac{\vartheta_{21}}{E_{22}} \tag{3.6}$$

To obtain the elastic constant matrix for any arbitrary principle axes with which the material principal axes makes an angle θ, standard coordination matrix transformation is required. Thus the off-axis elastic contant matrix is obtained from the on-axis elastic constant matrix as

$$[\bar{Q}_{ij}] = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} & 0 & 0 \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} & 0 & 0 \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} & 0 & 0 \\ 0 & 0 & 0 & \bar{Q}_{44} & \bar{Q}_{45} \\ 0 & 0 & 0 & \bar{Q}_{45} & \bar{Q}_{55} \end{bmatrix} \tag{3.7}$$

$$[\bar{Q}_{ij}] = [T]^T][Q_{ij}][T] \tag{3.8}$$

Where T is the transformation matrix. After transformation the elastic stiffness coefficients are.

$$\bar{Q}_{11} = Q_{11}m^4 + 2(Q_{12} + 2Q_{66})m^2n^2 + Q_{22}n^4$$

$$\bar{Q}_{12} = (Q_{11} + Q_{22} - 4Q_{66})m^2n^2 + Q_{12}(m^4 + n^4)$$

$$\bar{Q}_{22} = Q_{11}n^4 + 2(Q_{12} + Q_{66})m^2n^2 + Q_{22}m^4$$

$$\bar{Q}_{16} = (Q_{11} - Q_{12} - 2Q_{66})nm^3 + (Q_{12} - Q_{22} + 2Q_{66})n^3m$$

$$\bar{Q}_{26} = (Q_{11} - Q_{12} - 2Q_{66})mn^3 + (Q_{12} - Q_{22} + 2Q_{66})m^3n$$

$$\bar{Q}_{66} = (Q_{11} + Q_{22} - 2Q_{12} - 2Q_{66})n^2m^2 + Q_{66}(n^4 + m^4) \tag{3.9}$$

The elastic constant matrix corresponding to transverse shear deformation is

$$\bar{Q}_{44} = G_{13}m^2 + G_{23}n^2$$

$$\bar{Q}_{45} = (G_{13} - G_{23})mn$$

$$\bar{Q}_{55} = G_{13}n^2 + G_{23}m^2 \tag{3.10}$$

Where m = cosθ and n = sinθ

The stress strain relations are

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \\ \tau_{xz} \\ \tau_{yz} \end{bmatrix} = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} & 0 & 0 \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} & 0 & 0 \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} & 0 & 0 \\ 0 & 0 & 0 & \bar{Q}_{44} & \bar{Q}_{45} \\ 0 & 0 & 0 & \bar{Q}_{45} & \bar{Q}_{55} \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \\ \gamma_{xz} \\ \gamma_{yz} \end{bmatrix} \tag{3.11}$$

The forces and moment resultants are obtained by integration through the thickness h for stresses as

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_Y \\ M_{xY} \\ Q_x \\ Q_Y \end{bmatrix} = \int_{-h/2}^{h/2} \begin{Bmatrix} \sigma_x \\ \sigma_x \\ \tau_{xy} \\ \sigma_x z \\ \sigma_{yz} \\ \tau_{xy} z \\ \tau_{xz} \\ \tau_{yz} \end{Bmatrix} dz \tag{3.12}$$

Where σ_x, σ_y are the normal stresses along X and Y direction, τ_{xy},τ_{xz} and τ_{yz} are shear stresses in xy , xz and yz planes respectively.

Considering only in-plane deformation, the constitutive relation for the initial plane stress analysis is

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{21} & A_{22} & A_{26} \\ A_{31} & A_{32} & A_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} \tag{3.13}$$

The constitutive relationships for bending transverse shear of a doubly curved shell becomes

$$\begin{Bmatrix} N_x \\ N_P \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \\ Q_x \\ Q_y \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{112} & B_{16} & 0 & 0 \\ A_{21} & A_{22} & A_{26} & B_{12} & B_{22} & B_{26} & 0 & 0 \\ A_{16} & A_{26} & A_{66} & B_{11} & B_{12} & B_{16} & 0 & 0 \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} & 0 & 0 \\ B_{12} & B_{22} & B_{26} & D_{12} & D_{22} & D_{26} & 0 & 0 \\ B_{16} & B_{26} & B_{66} & D_{16} & D_{26} & D_{66} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & S_{44} & S_{45} \\ 0 & 0 & 0 & 0 & 0 & 0 & S_{45} & S_{55} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \\ k_x \\ k_y \\ k_{xy} \\ \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix} \tag{3.14}$$

This can also be stated as

$$\begin{Bmatrix} N_i \\ M_i \\ Q_i \end{Bmatrix} = \begin{bmatrix} A_{ij} & B_{ij} & 0 \\ B_{ij} & D_{ij} & 0 \\ 0 & 0 & S_{ij} \end{bmatrix} \begin{Bmatrix} \varepsilon_j \\ k_j \\ \gamma_m \end{Bmatrix} \tag{3.15}$$

Or $\{F\} = [D]\{\varepsilon\}$ (3.16)

Where A_{ij},B_{ij},D_{ij} and S_{ij} are the extensional , bending-stretching coupling, bending and transverse shear stiffness.

They may be defined as:

$$A_{ij} = \sum_{k=1}^{n} \overline{(Q_{ij})}_k (z_k - z_{k-1})$$

$$B_{ij} = \frac{1}{2} \sum_{k=1}^{n} \overline{(Q_{ij})}_k (z_k^2 - z_{k-1}^2)$$

$$D_{ij} = \frac{1}{3} \sum_{k=1}^{n} \overline{(Q_{ij})}_k (z_k^3 - z_{k-1}^3)\,; i,j = 1,2,6$$

$$S_{ij} = k \sum_{k=1}^{n} \overline{(Q_{ij})}_k (z_k - z_{k-1})\,; i,j = 4,5 \tag{3.17}$$

And k is the transverse shear correction factor.

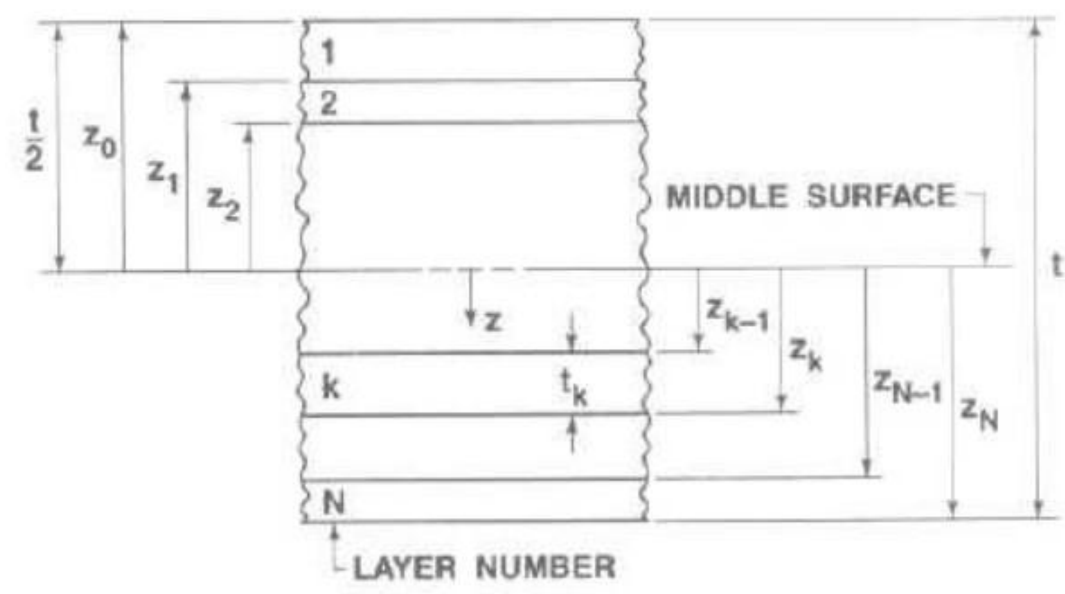


Fig 5: Geometry of a *N*-layered laminate

3.6 ANSYS METHODOLOGY

The software ANSYS was used to carry out the finite element analysis in the work. ANSYS is used to analyze the critical buckling load. Eigen value buckling analysis in ANSYS has four steps:

1. Build the model: It includes defining element type, real constants, material properties, sectioning, modelling and meshing. In this study shell, Elastic 8 node 281 is selected as the element type.

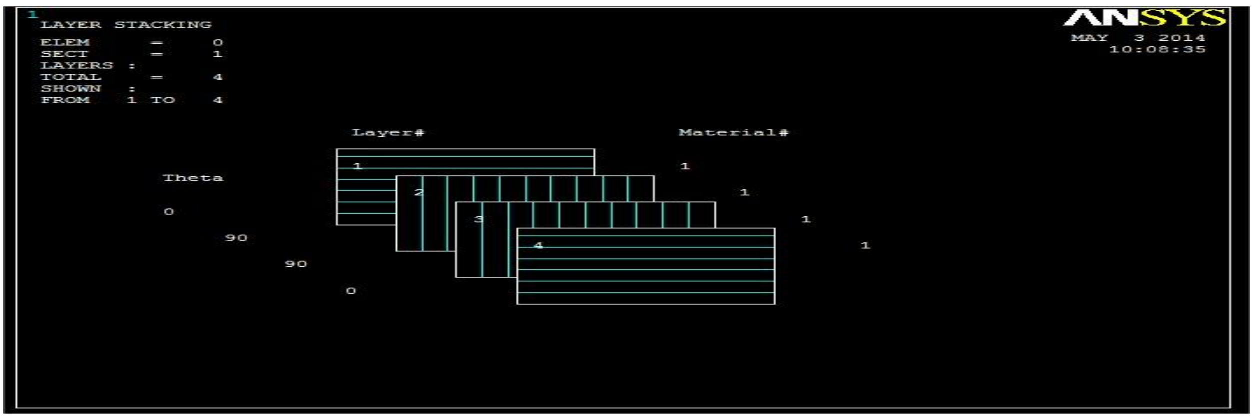


Fig 6: Lay-up

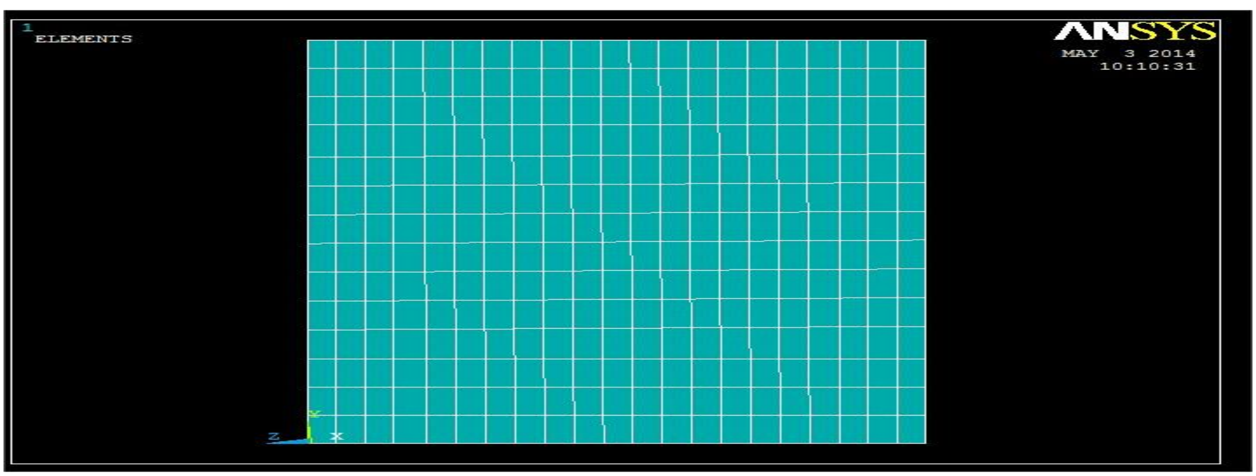


Fig 7: Meshing of plate

2. Solution (Static Analysis): It includes applying boundary conditions, applying loads and solving the static analysis. The applied boundary condition and load is shown below.

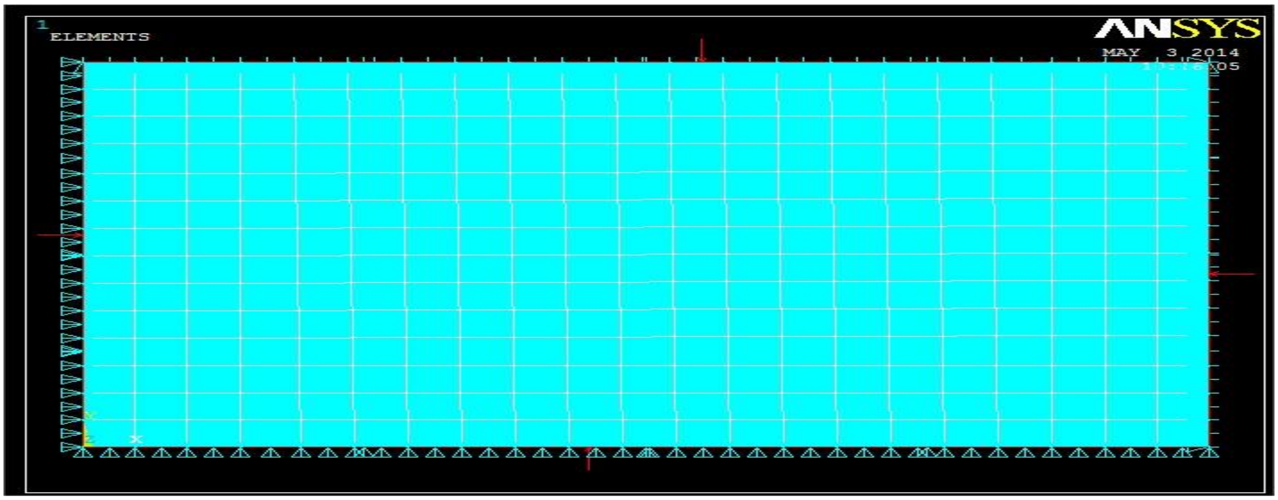


Fig 8: Loading and Boundary Condition

3. Eigen buckling analysis: Eigenvalue buckling analysis predicts the theoretical buckling strength of an ideal linear elastic structure.

4. Postprocessor: This step includes listing buckling loads and viewing buckled shapes. We can plot the deformed and undeformed shape of the plate.

CHAPTER 4

RESULTS AND DISCUSSION

Buckling behavior of laminated composite plates subjected to in-plane loads is an important aspect in the preliminary design of aircraft and launch vehicle components. These laminated composite plates may have holes placed centrally or otherwise for the purpose of pipes for electric cables or other purposes. This may reduce the stiffness of the plate and create stress concentrations in the region of the holes. Hence the study is important in order to know the buckling behavior of such plates. The aim of this project is to study the effect on buckling load of square and rectangular plates with and without a hole subjected to compressive loads with different boundary conditions.

4.1 CONVERGENCE STUDY

The following analysis is carried out to finalize the size of mesh. The buckling behavior of an orthotropic laminated composite plate having length 300mm and width 200mm is studied and results validated with those of Zhong and Gu [12]. The h/b ratio is fixed at 0.01. Hence the thickness works out to be 2mm. It has 3 layers oriented at 0, 90 and 0 degrees. Here ANSYS is used to find out the critical buckling load.

The material properties of the plate worked upon is as follows:

$E_T = E_H = 9.2\text{x}10^9 GPa$, $G_{xy} = G_{xz} = 5.52\text{x}10^9 GPa$, $G_{yz} = 4.6\text{x}10^9 GPa$, $\nu_{xy} = \nu_{xz} = 0.25$, $\nu_{yz} = \nu_{xy}/(E_L/E_T)$. E_L is found out from the ratio as specified in the paper.

The buckling load factor for a lay-up configuration [0/90/0] and h/b ratio = 0.01 is 22.048 according to Zhong and Gu[12].

Table 1: Convergence study

Mesh Size	Buckling Load (N/m)	Buckling Load Factor
4x4	41365	21.65
8x8	41565	21.88
12x12	41585	22.06

Hence at the final mesh size of 12x12 the results converge as can be seen from Table 1. Hence this mesh size is taken for further analysis.

4.2 VALIDATION OF PRESENT FORMULATION

Comparison of non-dimensional buckling load factors for symmetric cross-ply square plates subjected to uniaxial uniform loads and h/b = 0.1 is done with the results of Zhong and Gu [12]. Using the formula $k = Nb^2/E_T h^3$, buckling load factor is found out.

[where k- buckling load factor,

N- buckling load obtained from analysis,

b- width of the plate, in my analysis b=200 mm

E_T- Young's modulus in yz plane, value taken as 9.2e9

h- thickness of the plate, in my analysis h=2mm]

Table 2: Comparison of Results

E_L/E_T ratio	Lay-up Configuration [0/90/0]	
	Present Study	Zhong, Gu(2007)[12]
20	14.47	14.836
30	18.48	18.820
40	22.06	22.048

As can be observed from Table 2, the results of the study conducted in the project work are validated by the work of Zhong and Gu[12].

Next, results of present formulation were compared to that of Baba [11]. In this comparison, buckling behavior of an orthotropic laminated composite plate having thickness 1.5mm and width 25mm is studied. The l/t ratio is fixed at 50. Hence the length works out to be 75mm. It has 8 layers oriented at [(0/90)2] in a symmetric sequence. The diameter of the hole is 5mm. Here too ANSYS is used to find out the critical buckling load.

The material properties of the plate are as follows:

$E_L = 39x10^9 GPa$, $E_T = E_H = 8.2x10^9 GPa$, $G_{xy} = G_{xz} = G_{yz} = 2.9x10^9 GPa$,

$\nu_{xy} = \nu_{xz} = \nu_{yz} = 0.29$

For the stacking sequence and the l/t ratio taken, the results are compared with and without hole and with different boundary conditions.

Buckling loads (N/m) for all composites with l/t ratio = 50 and $[(0/90)_2]_s$, with and without cutout:

Table 3: Validation of Results with and without hole

	PLATE WITHOUT HOLE		PLATE WITH HOLE	
Boundary conditions ⇩	ANSYS	Baba[11]	ANSYS	Baba[11]
Clamped-clamped	1302	1105	1092	913
Clamped-pinned	857.73	971	703.29	852
Pinned-pinned	359.64	366.72	289.78	305

The results are almost similar to each other as seen in Table 3. Hence the results of the study conducted in the project work are compared and validated by the work of Baba [11].

4.3 RESULTS

Having decided on the mesh size and validated the results, the formulation is carried out for the present problem using ANSYS software.

Variation of buckling load with length/thickness ratio

The material properties of the plate were taken as:

$E_{11} = E_{33} = 141$GPa, $E_{22} = 9.23$ GPa,

$G_{12} = G_{13} = 5.95$ GPa, $G_{23} = 2.96$ GPa,

$\nu_{xy} = \nu_{xz} = 0.313$, $\nu_{yz} = 0.313/(141/9.23) = 0.0205$

Dimension of the plate:

It is a square plate with length and breadth equaling 0.5m. The stacking sequence is [0°/90°]. Hence the number of layers is two. The length/thickness ratio is changed from 0.1 to 0.5 in steps of 0.1. The variation of buckling load is studied. The boundary condition is simply-supported.

Table 4: Effect of length/thickness ratio on buckling load

Plate number	Length (m)	L/H ratio	Thickness (m)	Buckling Load(N/m^2)
1	0.5	0.1	0.05	0.34876E+08
2	0.5	0.2	0.1	0.21719E+09
3	0.5	0.3	0.15	0.51416E+09
4	0.5	0.4	0.2	0.74966E+09
5	0.5	0.5	0.25	0.93845E+09

As the l/h ratio increases, the buckling load increases as seen from Table 4.

Mode Shapes

For L/H ratio = 0.5

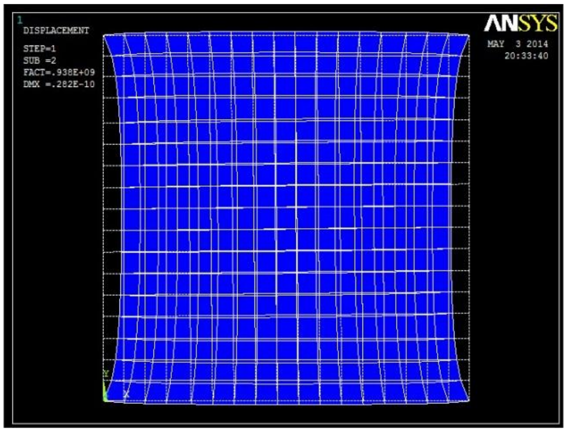


Fig 9(a): Deformed Shape

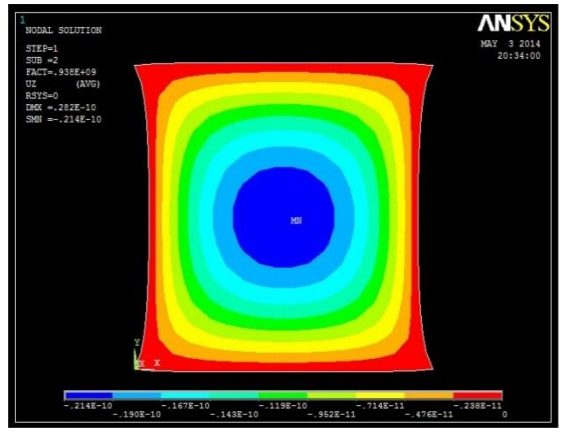


Fig 9(b): Z-displacement

Variation of buckling load with different boundary conditions:

Material Properties of the plate taken:

$E_{11} = E_{33} = 141$GPa, $E_{22} = 9.23$ GPa,

$G_{12} = G_{13} = 5.95$ GPa, $G_{23} = 2.96$ GPa,

$\nu_{xy} = \nu_{xz} = 0.313$, $\nu_{yz} = 0.313/(141/9.23) = 0.0205$

Dimension of the plate taken:

It is a square plate with length and breadth equaling 0.5m. The diameter of the hole is found from the d/b ratio which is fixed at 0.1. So the diameter comes out to be 50mm. Two stacking sequences are taken; [0/90] and [0/90/0]. The total thickness is taken to be 5mm. A central circular hole is taken for the analysis.

Boundary Conditions taken:

Three boundary conditions are taken; Clamped-free, simply-supported and fixed.

Table 5(a): Variation of buckling load with boundary conditions

[0°/90°] stacking sequence

Boundary conditions ⇩	Plate without hole(N/m)	Plate with hole(N/m)
Clamped-free	17.65	1.9202
Simply-supported	20181	19952
Fixed	82738	65461

Table 5(b): Variation of buckling load with boundary conditions

[0°/90°/0°] stacking sequence

Boundary conditions ⇩	Plate without hole(N/m)	Plate with hole(N/m)
Clamped-free	86.932	21.61
Simply-supported	41113	26656
Fixed	0.10656E+06	0.10479E+06

Tables 5 show that the fixed plate shows the largest value of buckling load as compared to simply supported and cantilever plates and the buckling load decreases with the presence of cut-out.

Analysis of buckling behavior with change in stacking sequence:

Table 6(a): Variation of buckling load with change in stacking sequence

Plate without hole

Boundary conditions ⇩	[0°/90°] (N/m)	[0°/90°/0°] (N/m)
Clamped-free	17.65	86.932
Simply-supported	20181	41113
Fixed	82738	0.10656E+06

Table 6(b): Variation of buckling load with change in stacking sequence

Plate with hole

Boundary conditions ⇩	[0°/90°] (N/m)	[0°/90°/0°] (N/m)
Clamped-free	1.9202	21.61
Simply-supported	19952	26656
Fixed	65461	0.10479E+06

Tables 6 show that for plates with or without holes the buckling load increases with number of layers.

Mode Shapes

Mode shape corresponding to lowest buckling load is shown below for the different cases. The variation in displacement in z-direction is also shown.

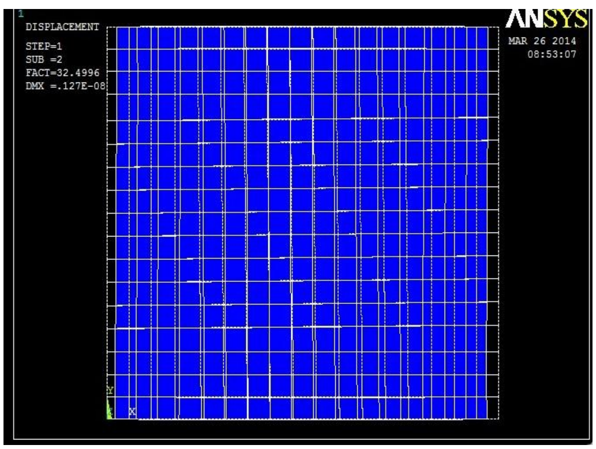


Fig 10(a): Deformed Shape

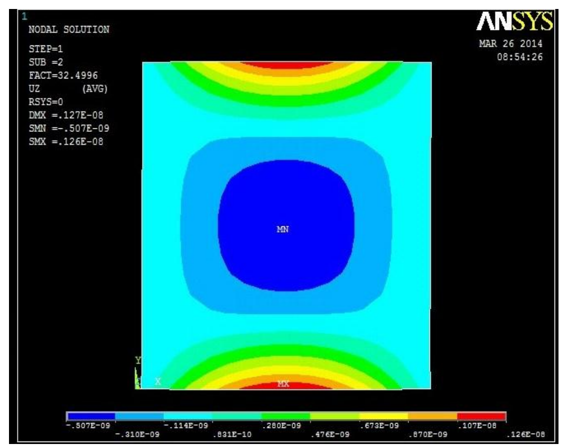
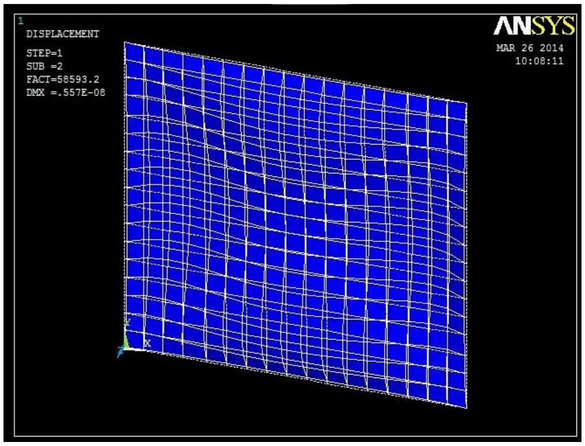


Fig 10(b): Z displacement

Fig 10: Plots for 2-layered [0°/90°] clamped-free composite plate without hole



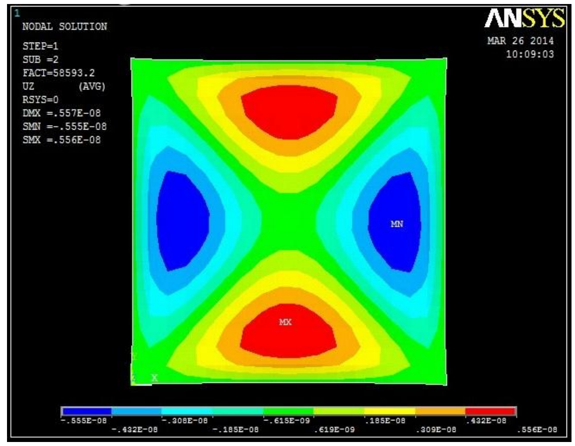
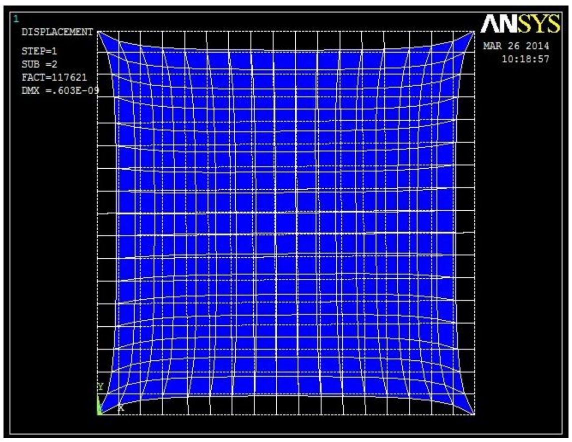


Fig 11(a): Deformed Shape

Fig 11(b): Z displacement

Fig 11: Plots for 2-layered [0°/90°] simply-supported composite plate without hole



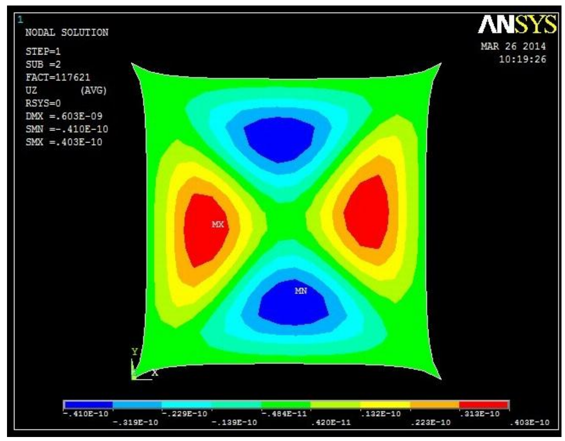
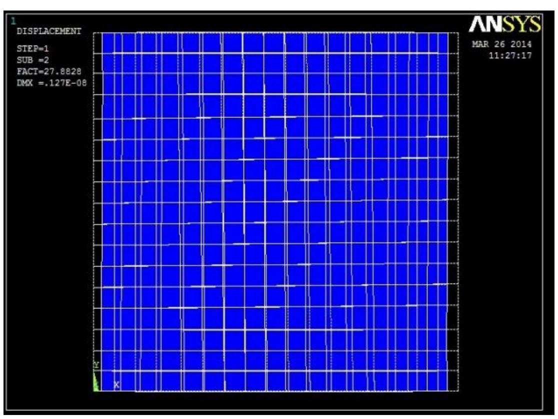


Fig 12(a): Deformed Shape

Fig 12(b): Z displacement

Fig 12: Plots for 2-layered [0°/90°] fixed composite plate without hole



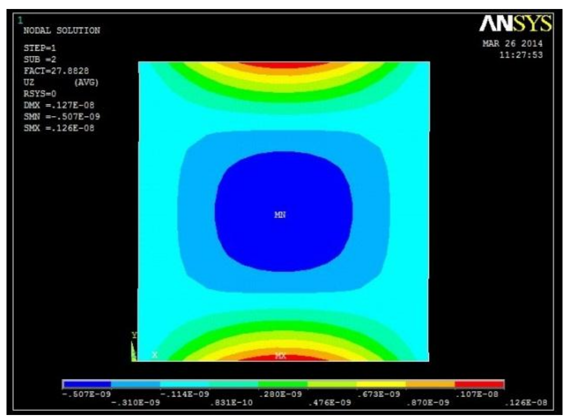


Fig 13(a): Deformed Shape

Fig 13(b): Z displacement

Fig 13: Plots for 3-layered [0°/90°/0°] clamped-free composite plate without hole

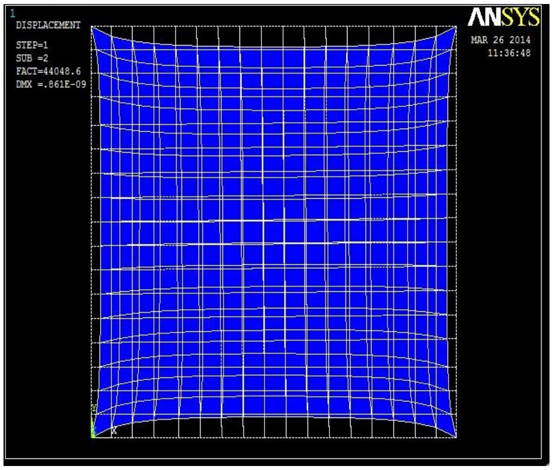


Fig 14(a): Deformed Shape

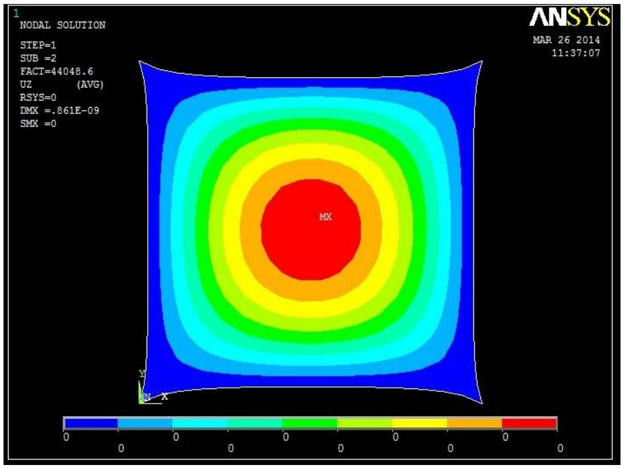


Fig 14(b): Z displacement

Fig 14: Plots for 3-layered [0°/90°/0°] simply-supported composite plate without hole

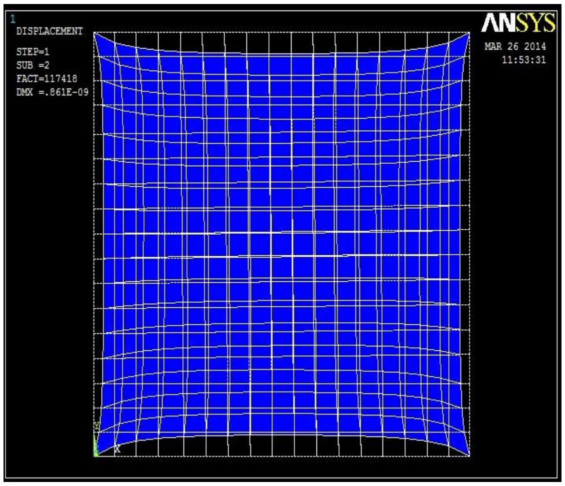


Fig 15(a): Deformed Shape

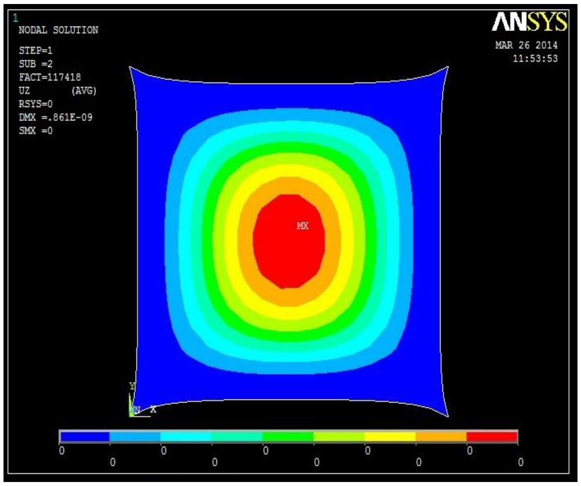


Fig 15(b): Z displacement

Fig 15: Plots for 3-layered [0°/90°/0°] fixed composite plate without hole

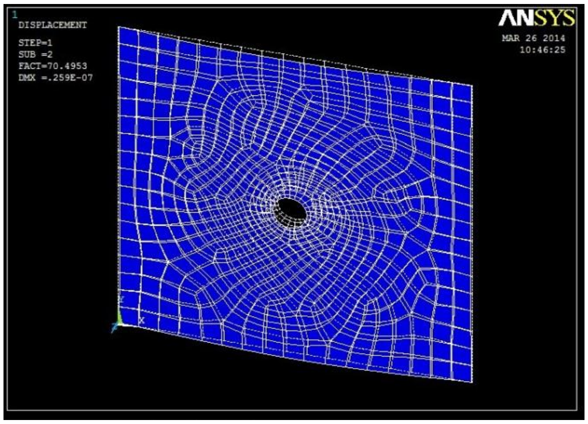


Fig 16(a): Deformed Shape

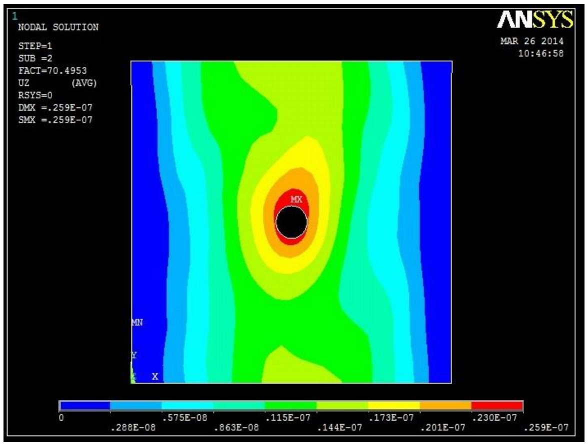
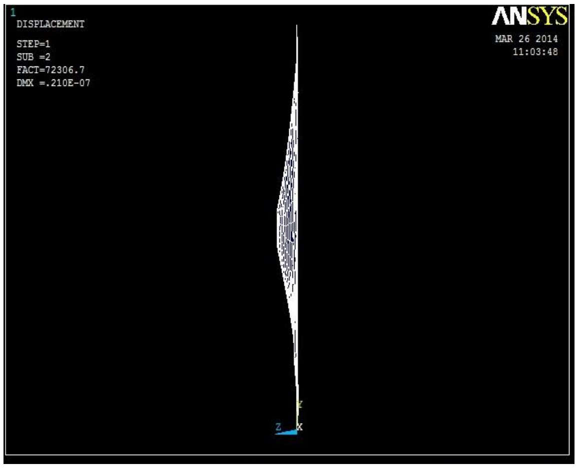


Fig 16(b): Z displacement

Fig 16: Plots for 2-layered[0°/90°] clamped-free composite plate with hole



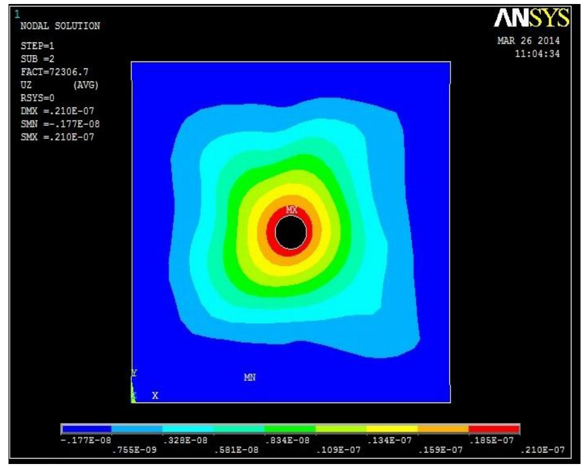
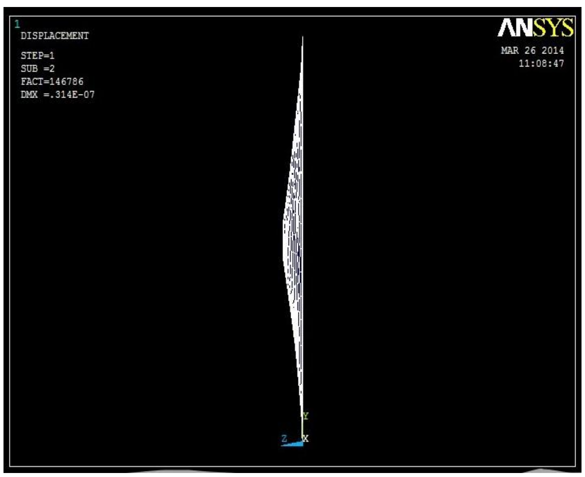


Fig 17(a): Deformed Shape

Fig 17(b): Z displacement

Fig 17: Plots for 2-layered [0°/90°] simply-supported composite plate with hole



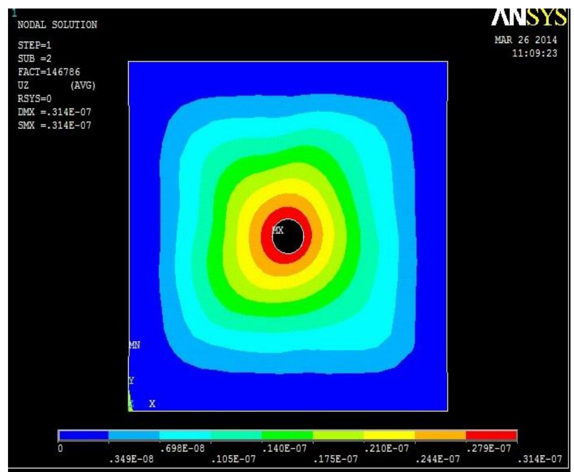
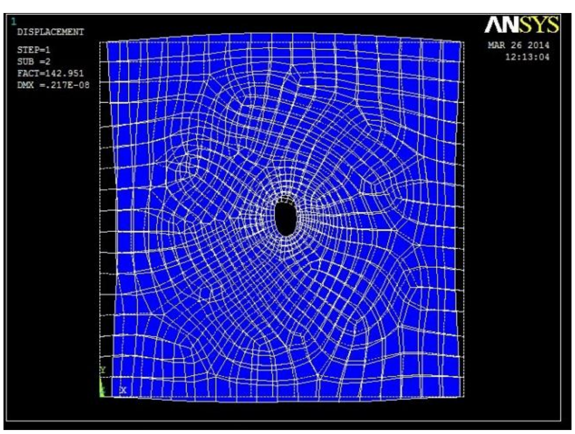


Fig 18(a): Deformed Shape

Fig 18(b): Z displacement

Fig 18: Plots for 2-layered [0°/90°] fixed composite plate with hole



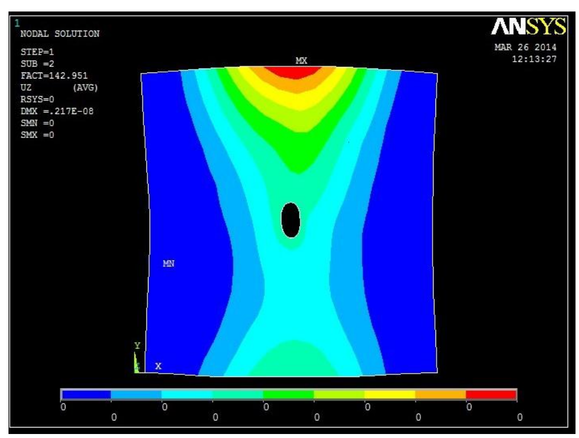


Fig 19(a): Deformed Shape

Fig 19(b): Z displacement

Fig 19: Plots for 3-layered [0°/90°/0°] clamped-free composite plate with hole

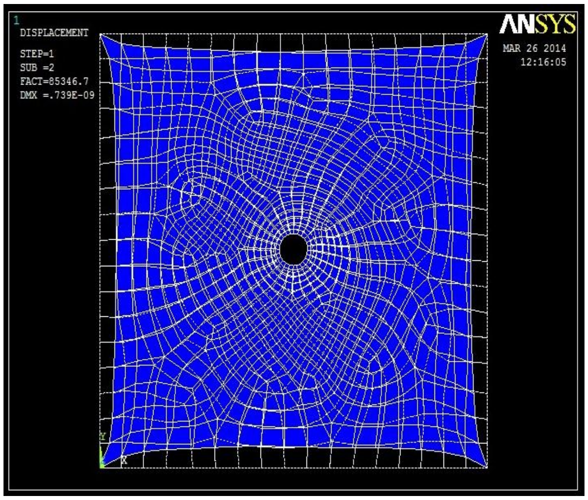


Fig 20(a): Deformed Shape

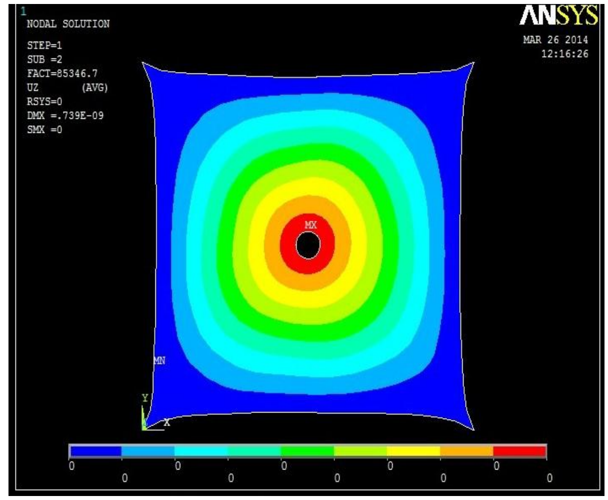


Fig 20(b): Z displacement

Fig 20: Plots for 3-layered [0°/90°/0°] simply-supported composite plate with hole

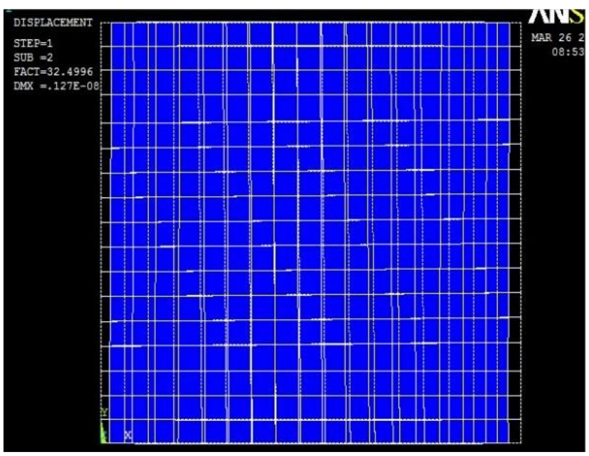


Fig 21(a): Deformed Shape

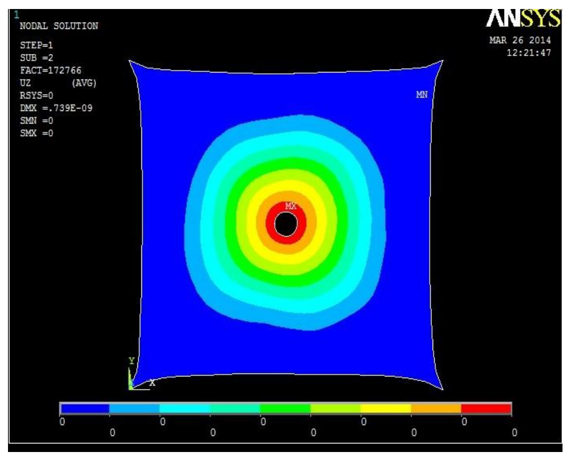


Fig 21(b): Z displacement

Fig 21: Plots for 3-layered [0°/90°/0°] fixed composite plate with hole

Variation of Buckling Load with shape of Cut-out

Material Properties of the plate taken:

$E_{11} = E_{33} = 141$GPa, $E_{22} = 9.23$ GPa,

$G_{12} = G_{13} = 5.95$ GPa, $G_{23} = 2.96$ GPa,

$\nu_{xy} = \nu_{xz} = 0.313$, $\nu_{yz} = 0.313/(141/9.23) = 0.0205$

Dimension of the plate taken:

It is a square plate with length and breadth equaling 0.5m. Stacking sequence taken is [0°/90°/0°]. The total thickness is taken to be 5mm. Three shapes of cut-outs are taken; circular, square and triangular. The dimensions of the cut-outs are same. The diameter of the circular cut-out is 50 mm. The triangular cut-out is equilateral with sides 50mm and the dimension of square cut-out is even 50mm. The boundary condition taken is simply-supported.

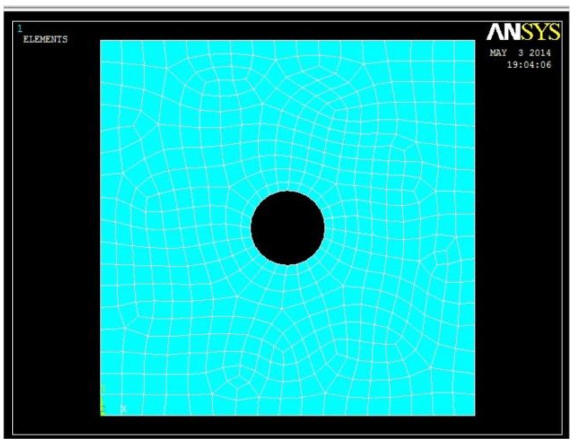


Fig 22: Model with circular cut-out

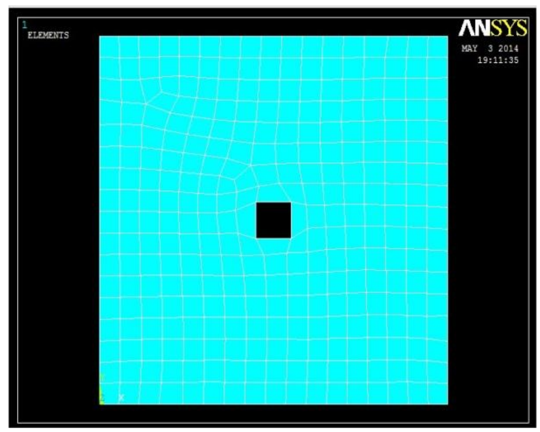


Fig 23: Model with square cut-out

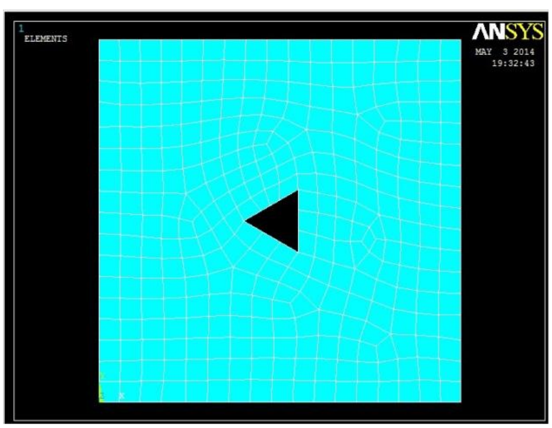


Fig 24: Model with triangular cut-out

Table 7: Effect of shape of cut-out on buckling load

Shape of Cut-out	Dimension of cut-out	Buckling Load(N/m)
Circular	50mm diameter	31617
Triangular	Equilateral triangle with sides 50mm	44834
Square	Sides 50mm	48193

The plate with square cutout shows the highest buckling load as compared to the circular and triangular cutout as seen in Table 7.

Mode Shapes

The mode shape for lowest buckling is shown below. The variation in displacement in z-direction is also shown.

Circular cut-out

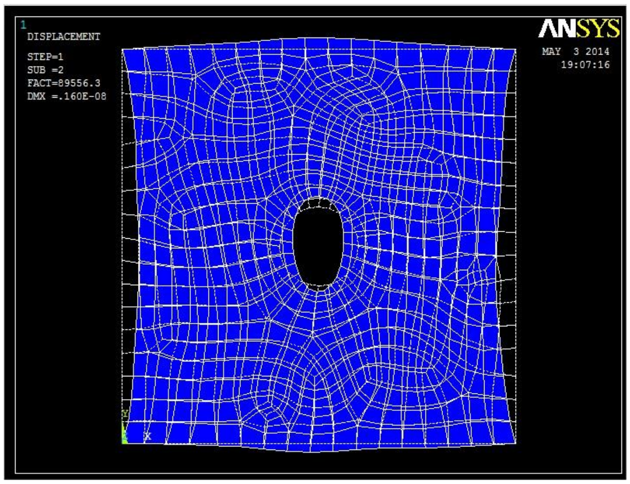


Fig 25(a): Deformed Shape

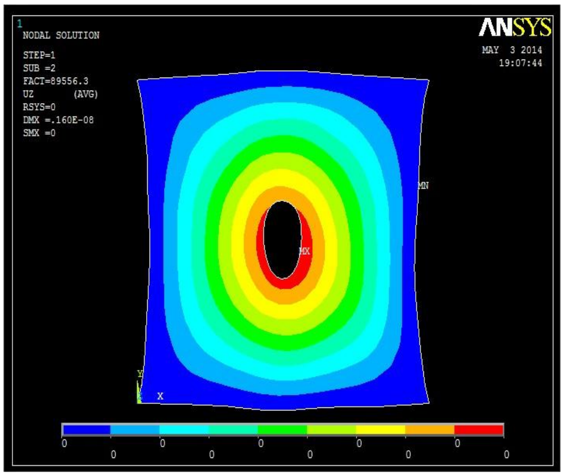


Fig 25(b): Z-displacement

Triangular cut-out

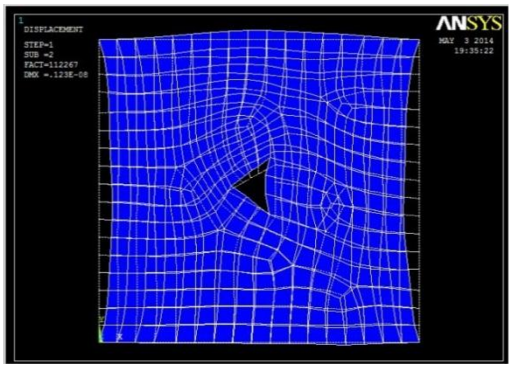


Fig 26(a): Deformed Shape

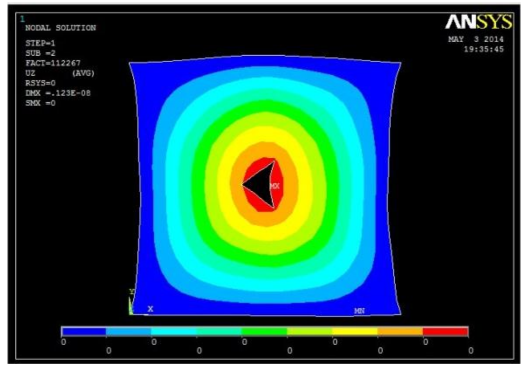


Fig 26(b): Z-displacement

Square cut-out

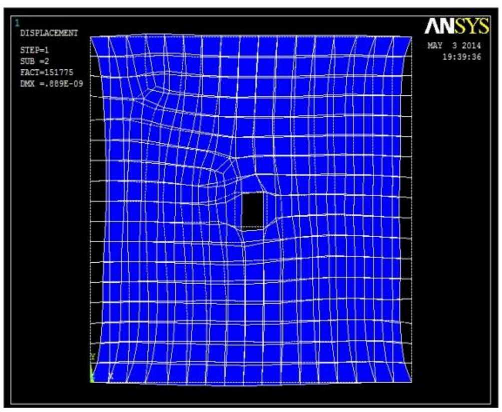


Fig 27(a): Deformed Shape

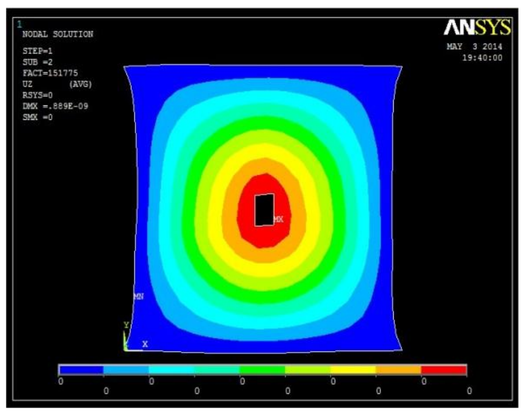


Fig 27(b): Z-displacement

Variation of Buckling Load with ply orientation

Material Properties of the plate taken:

$E_{11} = E_{33} = 141$GPa, $E_{22} = 9.23$ GPa,

$G_{12} = G_{13} = 5.95$ GPa, $G_{23} = 2.96$ GPa,

$\nu_{xy} = \nu_{xz} = 0.313$, $\nu_{yz} = 0.313/(141/9.23) = 0.0205$

Dimension of the plate taken:

It is a square plate with length and breadth equaling 0.5m. The cut-out diameter taken is 50mm. The number of layers taken is 4. The orientations taken [(15/-15)2] s, [(30/-30)2] s, [(45/-45)2]s, and [(60/-60)2]s. The boundary condition taken is simply-supported.

Table 8: Effect of ply oriention on buckling load

Ply Orientation	Buckling Load (N/m)
[15,-15,-15,15]	55390
[30,-30,-30,30]	88846
[45,-45,-45,45]	0.11176E+06
[60,-60,-60,60]	0.12368E+06

The buckling load increases as the angle of orientation of the ply increases as seen in Table 8.

Mode Shapes

For Ply Orientation [(60,-60)2] s

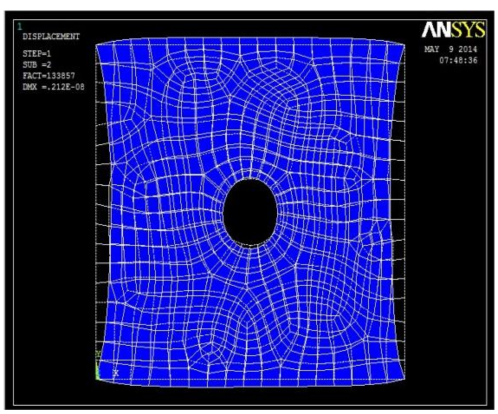


Fig 28(a): Deformed Shape

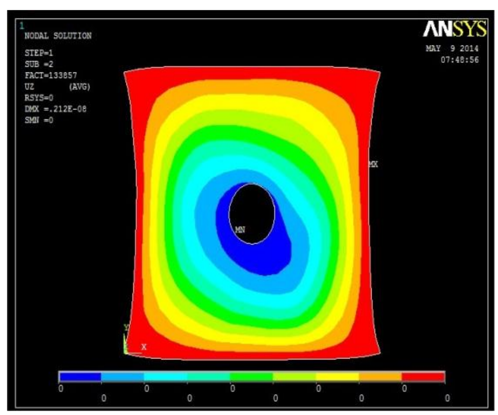


Fig 28(b): Z-Displacement

CHAPTER 5

CONCLUSION

5.1 CONCLUSION

From the analysis carried out by the use of ANSYS on the laminated composite plates, the following conclusions have been drawn out.

1. As the E_L/E_T ratio increases, the buckling load factor and hence, the buckling load increases, as can be seen from the validation work done.

2. The increase in length/thickness ratio increases the buckling load.

3. Due to presence of cut-out, the buckling load decreases. As the surface area decreases in presence of cut-out, the load required to buckle the plate and deform its shape becomes less. Hence the buckling load decreases.

4. As the number of layers increases, the buckling load also increases. This is because as the number of layers increases, the interaction between each layer increases and therefore high amount of load is required to get the critical buckling load.

5. The buckling load changes with change in boundary condition. The buckling load for clamped-free plate is least; followed by simply-supported plate and for fixed support plates, the buckling load is the highest.

6. Buckling load changes with change of cut-out shapes as well. For circular cut-outs, the buckling load comes out to be the least. And for square cut-outs of same dimension, the buckling load is maximum. Triangular cut-outs have buckling load intermediate between circular and square.

7. Buckling load increases as the angle in the ply orientation increases.

5.2 FUTURE SCOPE OF WORK

In the present study, the effect of presence of cut-outs, change of shapes of cut-outs, change of modulus ratio, change of boundary condition, change of ply orientation and change of stacking sequence has been studied. The future scope of the present investigation is as follows:

1) The laminated composites can be subjected to bi-axial compressive loading and the effect on buckling behavior can be studied.

2) The plate can be subjected to shear loading too and the buckling behavior can be studied extensively.

3) The plates can be subjected to temperature variation and thus a thermal stress can be developed which can be studied.

CHAPTER 6

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